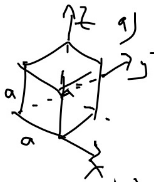
Fizika I gyakorlat 6

F1



a) $\underline{E} = E_0 \hat{z}$ $\qquad \Phi_E$

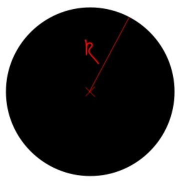
sík		$\underline{n}$	
xy	$z=0$	$\underline{n}=(0,0,-1)$	$\Phi = -E_0 a^2$
	$z=a$	$\underline{n}=(0,0,1)$	$E_0 a^2$
xz	0	$(\underline{n} \perp \underline{E})$	
yz	0		

b) $\underline{E} = \frac{E_0}{\sqrt{3}}(\underline{e}_x + \underline{e}_y + \underline{e}_z)$

$$\Phi_{xy}^{(z=0)} = \underline{E}\cdot a^2 \cdot (0,0,-1) = -\frac{E_0}{\sqrt{3}} a^2$$
$$(0,0,1) = +\frac{E_0}{\sqrt{3}} a^2$$
$$\Phi_{xz} = \frac{E_0}{\sqrt{3}} a^2 \cdot (0,-1,0)$$
$$(0,1,0)$$

c) Q a középpontban $\qquad \Phi_{tot} = \frac{Q}{\varepsilon_0} \qquad \Phi_{lap} = \frac{1}{6}\frac{Q}{\varepsilon_0}$

F2

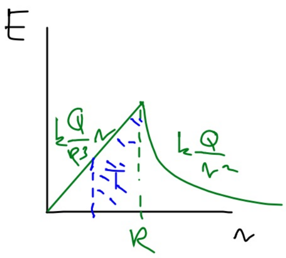


Q $\quad \rho = \text{áll}$

a) $\underline{E}(\underline{r})$

b) $V(\underline{r})$

c) ugyanez fémg.



a) $r > R \qquad \underline{E} = k\frac{Q}{r^2}\frac{\underline{r}}{r}$

$$r < R \Rightarrow Q(r) = \frac{4\pi}{3}r^3 \cdot \rho = \cancel{\frac{4\pi}{3}}r^3 \cdot \frac{Q}{\cancel{\frac{4\pi}{3}}R^3} = \frac{Q}{R^3}r^3$$

$$E(r) = k\frac{Q(r)}{r^2}\frac{r}{r} = k\frac{Q}{R^3}\cdot\frac{\cancel{r^3}}{\cancel{r^2}} = k\frac{Q}{R^3}r$$

$$T = \frac{E(r)+E(R)}{2}(R-r) = \frac{1}{2}\left(\frac{Q}{R^3}r + \frac{Q}{R^3}R\right)(R-r) = \frac{1}{2}\frac{Q}{R^3}(r+R)(R-r) = \frac{1}{2}\frac{Q}{R^3}(R^2-r^2)$$

b) $V(r) \quad V(\infty) = 0$

$$V(r) = k\frac{Q}{r} \qquad r > R$$

$$V(r) = k\frac{Q}{R} - \int_R^r k\frac{Q}{R^3}r\,dr = k\frac{Q}{R} - \frac{kQ}{2R^3}\left[r^2\right]_R^r$$

$$= k\frac{Q}{R}\left(1 - \frac{1}{2}\frac{r^2-R^2}{R^2}\right) = k\frac{Q}{R^3}\left(R^2 + \frac{1}{2}R^2 - \frac{1}{2}r^2\right)$$

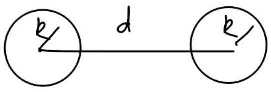
$$V(r) = \frac{kQ}{2R^3}(3R^2 - r^2)$$

c) fémgömb

$$\boxed{E(r) = \begin{cases} k\frac{Q}{r^2} & r \geq R \\ 0 & r \leq R \end{cases} \qquad V(r) = \begin{cases} k\frac{Q}{r} & r \geq R \\ k\frac{Q}{R} & r < R \end{cases}}$$

F3

$d(t=0) = 12\,cm$



$Q_1 = -10\,nC$ $\quad$ $Q_2 = 20\,nC$

$m_1 = 10\,g$ $\quad$ $m_2 = 5\,g$

$v(ütk) = ?$

$p = áll = 0$

$0 = m_1 v_1 + m_2 v_2$

$-\frac{m_1}{m_2} v_1 = v_2 \implies$

$E(t=0) = E_{pot} = k\frac{Q_1 Q_2}{d}$

$E(ütk) = k\frac{Q_1 Q_2}{2R} + \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$

$E = áll$

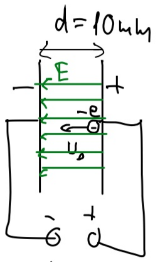
$$k\frac{Q_1 Q_2}{2R} + \frac{1}{2}\left(m_1 v_1^2 + \frac{m_1^2}{m_2} v_1^2\right) = k\frac{Q_1 Q_2}{d}$$

$$\underbrace{kQ_1Q_2}_{<0}\underbrace{\left(\frac{1}{d} - \frac{1}{2R}\right)}_{<0} = \frac{1}{2}\left(m_1 + \frac{m_1^2}{m_2}\right) v_1^2$$

$$v_1 = \sqrt{\frac{kQ_1Q_2\left(\frac{1}{d} - \frac{1}{2R}\right)}{\frac{1}{2}\left(m_1 + \frac{m_1^2}{m_2}\right)}} = 0{,}045\,m/s$$

$$v_2 = -\frac{m_1}{m_2} v_1 = -0{,}09\,m/s$$

F4



$\mathcal{E}_{kin}(0) = 9\,eV$

a) max távolság (s)

b) $v_0 = ?$

c) $a = ?$

$E = \frac{U}{d}$ $\quad$ $F = e\frac{U}{d}$ $\quad$ $a = \frac{eU}{m_e d}$

a) $\mathcal{E}_{kin}(0) = eE\cdot s$

$\mathcal{E}_{kin}(0) = \frac{eU}{d}\cdot s$

$9\cancel{eV} = \frac{12\cancel{eV}}{10\,mm}\cdot s$

$s = \frac{90}{12}\,mm = 7{,}5\,mm$

b) $\mathcal{E}_{kin}(0) = \frac{1}{2} m_e v_0^2$

$$v_0 = \sqrt{\frac{2\mathcal{E}_{kin}(0)}{m_e}} = \sqrt{\frac{2\cdot 9\,eV\cdot 1{,}6\cdot 10^{-19}\,\frac{J}{eV}}{9{,}1\cdot 10^{-31}\,kg}} = 1{,}77\cdot 10^{6}\,\frac{m}{s}$$

c) $a = \frac{eU}{m_e d} = 2{,}1\cdot 10^{14}\,m/s^2$

F5

$\varphi(\underline{r}) = \alpha x^2 + \beta$, $\alpha = 3\,V/m^2$ $\beta = 7V$ elektron

$\underline{r}_0 = (1,0,0)$

a) 2m után $\mathcal{E} = ?$ eV

b) $v = ?$

c) $a(t=0) = ?$

$$\text{a) } \Delta\mathcal{E} = \mathcal{E}(2m) - \mathcal{E}(0m) = -e\Delta\varphi = -e\left(\varphi(x_0+2) - \varphi(x_0)\right) \qquad x_0 = 1m$$

$$= -e\left(\alpha\cdot(3m)^2 + \cancel{\beta} - \alpha(1m)^2 - \cancel{\beta}\right) = -e\left(3\frac{V}{m^2}\cdot 8m^2\right) = -\underline{e}\,24V$$

J-ban $\Leftarrow$ $\underline{\underline{-24eV}}$

irány? $\underline{E} = -\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right) = -(2\alpha x, 0, 0)$ negatív x irányba mutat

, erő $= -e\underline{E}$ pozitív x irányba mutat

b) munkatétel $\Rightarrow$ $\Delta\mathcal{E} = \Delta\mathcal{E}_{kin}$

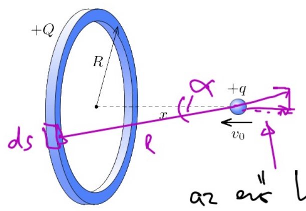
$$24eV = \frac{1}{2} m_e v^2 \Rightarrow$$

$$\underline{\underline{v}} = \sqrt{\frac{48eV}{9{,}1\cdot 10^{-31}kg}} = \sqrt{\frac{48\cancel{eV}\cdot 1{,}6\cdot 10^{-19}\,J/\cancel{eV}}{9{,}1\cdot 10^{-31}kg}} = \underline{\underline{2{,}9\cdot 10^{6}\frac{m}{s}}}$$

$$\text{c) } \underline{\underline{a(t=0)}} = \frac{F(t=0)}{m_e} = \frac{F(x_0)}{m_e} = -\frac{eE(x_0)}{m_e} =$$

$$= -\frac{1{,}6\cdot 10^{-19}C\cdot(-6V)}{9{,}1\cdot 10^{-31}kg} = \underline{\underline{1{,}1\cdot 10^{12}\frac{m}{s^2}}}$$

F6



$v_0 = ?$ $\Rightarrow$ áthaladjon

taszító erő

dQ

az erő helyett a potenciális energiával érdemes

$$q\,d\varphi(x) = \underbrace{k\frac{qQ}{2R\pi}\cdot ds}_{dQ}\cdot\frac{1}{\ell(x)} \qquad \ell(x) = \sqrt{x^2+R^2}$$

$$\varphi(x) = \Sigma\, d\varphi(x) = 2R\pi\cdot d\varphi(x) = k\frac{Q}{\ell(x)} = k\frac{Q}{\sqrt{x^2+R^2}}$$

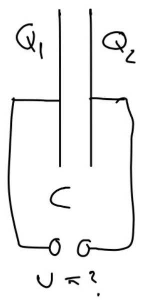
ha épp megáll $\Rightarrow$ $\frac{1}{2}mv_0^2 = q\left(\varphi(0) - \varphi(x)\right) = q\cdot\left(\varphi(0) - \varphi(2R)\right)$

$$= k\cdot Qq\cdot\left(\frac{1}{R} - \frac{1}{\sqrt{5}R}\right)$$

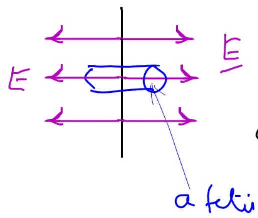
$$\boxed{V_0=\sqrt{\frac{2kQq}{mR}\cdot\left(1-\frac{1}{\sqrt{5}}\right)}=\sqrt{\frac{kQq}{mR}\,\frac{2(\sqrt{5}-1)}{\sqrt{5}}}=\sqrt{\frac{kQq}{mR}\,\frac{2(5-\sqrt{5})}{5}}}$$

bármelyik jó eredmény

F7



egy lemez, Gauss tétellel



A – lemez területe

$\sigma = \frac{Q}{A}$

Gauss: $E_i \cdot 2\cancel{a} = \frac{1}{\varepsilon_0}\cancel{a}\,\frac{Q_i}{A}$ $\quad i=1,2$

$E_i = \frac{Q_i}{2\varepsilon_0 A}$

lemezek között $E = E_1 - E_2$

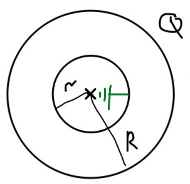
$U = E\cdot d$ $\quad$ d a lemezek távolsága

$$U=\frac{1}{2\varepsilon_0 A}\cdot d\,(Q_1-Q_2)$$

síkkondenzátor:

$$C=\varepsilon_0\frac{A}{d} \Rightarrow \boxed{U=\frac{Q_1-Q_2}{2C}}$$

F8



földelés ⇒ ∀ töltés vándorolhat a földelésen keresztül a ∞-hez képest

$\varphi(Q)=k\frac{Q}{R}$ a felületen és belül (lsd F2)

a „föld" potenciálja 0 ⇒ r-en belül $\varphi=0$

$\Downarrow$

R és r között $\varphi = k\frac{Q}{R}$

r-en belül $\varphi = k\frac{q}{r}$ $\quad \Rightarrow \quad k\frac{q}{r}+k\frac{Q}{R}=0$

$$\boxed{q=-Q\frac{r}{R}}$$